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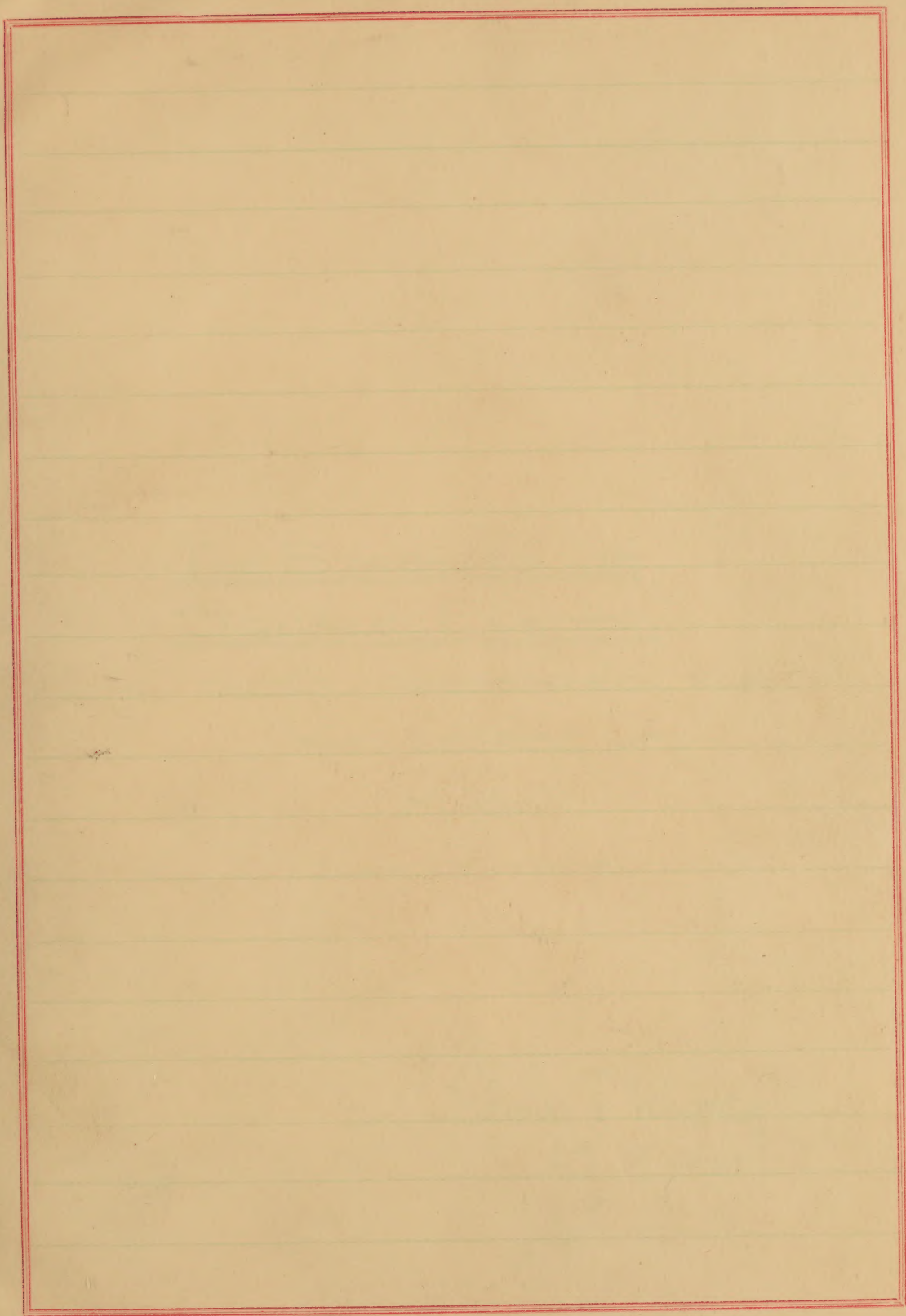
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THE STRENGTH OF  
SQUARE PLATES.

THESIS PRESENTED FOR  
THE DEGREE OF  
M. S.

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Lewis F. Moody  
B. S. 1901.



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## The Strength of Square Plates.

In designing a square plate to be subjected to a uniform pressure over its surface, its edges to be held by bolts, there is no definite formula to be found which will even approximately give the thickness of metal required. The problem of finding the thickness of such a plate frequently arises in machine design. While it might seem at first sight that the problem is simple enough to be solved by rational methods, nevertheless the fact remains that there is no satisfactory formula for the case; although it has received attention from Grashof, St. Venant, Clebsch, Bach, Sanza, and others. The practical form of the problem - as it arises in regard to valve-chest covers and condenser-zuds, for

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instance - is that of the thickness required for a plate which is neither simply supported around its edge without any bending moment being applied, nor rigidly fixed or "encastré" at every point of its boundary. If the plate were strictly encastré, the tangent to its surface, when it is bent under a load, could be considered horizontal at the boundary. But in actual plates, held with bolts, the degree of restraint is not perfect, and the condition of the plate lies somewhere between that of being simply supported, and that of being fixed.

In a round plate the reaction at every point in the supporting ring is the same, and is known, but in a square or rectangular plate the reaction varies from point to point. These considerations show a part of the difficulties met with at the very start. As far as I have





been able to ascertain, no rational formula exists for plates encastré.

This is a definite problem, and will probably be solved. Much doubt exists in regard to supported plates; and without experimental confirmation, I should not accept a formula for this case, though it is the simplest one, for square plates under uniform pressure.

The literature on the subject is limited in extent and abstruse in substance; most of the work has been done by the German investigators, and is untranslated.

The formula usually quoted in that book is that of Bressler; and it is generally recognized that this formula gives unnecessarily thick plates, when practically applied.

The consideration of the subject from the theoretic side is exceedingly involved





and obscure, and it may be said without exaggeration, that the results of all the discussions, founded on the Theory of Elasticity which I have seen, either cannot be practically applied; or else, when applicable to concrete cases, they give absurd values. This is due to the fact that the elastic laws do not hold, for material stressed above the elastic limit; and besides this, in most of the discussions, inadmissible assumptions are made at the start.

The one useful conclusion which we may take from these discussions is the fact - which may be accepted without going into the proof at present - that the unit stress at the middle of the plate varies directly as the total pressure on the plate and inversely as the square of the thickness, so that calling the thickness  $h$ , we have:  
$$S \propto \frac{p}{h^2}; \text{ or, } h = C a \sqrt{\frac{P}{S}};$$





where  $a$  = length of one side of the plate (in inches);  $p$  = unit pressure (in pounds per square inch);  $S$  = unit stress at the center of the plate (in pounds per square inch) - breaking being assumed to occur at the center first; - and  $C$  = a constant, depending on the method of supporting the plate, and determined by experiment.

Depending upon theory for the form of the equation, the object of this thesis may be stated to be the determination of an empirical equation for the thickness of square plates, in terms of their dimensions and the loads carried.



### Discussion.

Prof. Merriman, in his "Mechanics of Materials", a book which may usually be relied on for its accuracy, gives the following formula for square plates, which he claims to have taken from Bach:

$$S = \frac{9}{8} \frac{a^2 p}{d^2} \text{ or } S = \frac{3}{4} \frac{a^2 p}{d^2}$$
 (d is the thickness), according as the condition of the edges approached that of a mere support or a state of fixedness. But, on page 555 of "Elasticität und Festigkeit" by C. Bach, the following equation is given:

$$k_b \geq \frac{1}{4} \mu \left( \frac{a}{h} \right)^2 p$$
, where  $k_b$  is the 'allowable resistance against bending'. Bach states that the researches of experimenters give to  $\mu$  values varying from  $\frac{3}{4}$  to  $\frac{1}{2}$ . In Prof. Merriman's notation this gives:

$$S = \frac{1}{4} \cdot \frac{9}{8} \frac{a^2 p}{d^2} \text{ and } S = \frac{1}{4} \cdot \frac{3}{4} \frac{a^2 p}{d^2}$$
 ; showing that he has omitted the  $\frac{1}{4}$  in making the quotation. If the formula be put in





the form previously stated: -  $k_b = C \sqrt{\frac{E}{p}}$ , -  
( $C \in \frac{1}{\sqrt{p}}$ ) will vary between

$$C = \frac{\sqrt{24}}{2} = .433 \quad \text{and}$$

$$C = \frac{\sqrt{28}}{2} = .529,$$

the former value corresponding to fixed edges, and the latter to free support.

(As the plates I have tested have been bolted to the supports as tightly as possible, it is to be expected that the value of  $C$  will approach the first one given, and if the ultimate 'resistance against bending' or modulus of rupture of a specimen of the material broken as a beam, be put in the equation for  $k_b$  in 5, we should obtain a value for  $C$  between .433 and .529, and probably very close to .433, - the breaking pressure of the plate being inserted for  $p$ .

A brief discussion of the problem of a plate simply supported around its edges,





will be given, in the form used by Buse, to illustrate one way of attacking such a problem.

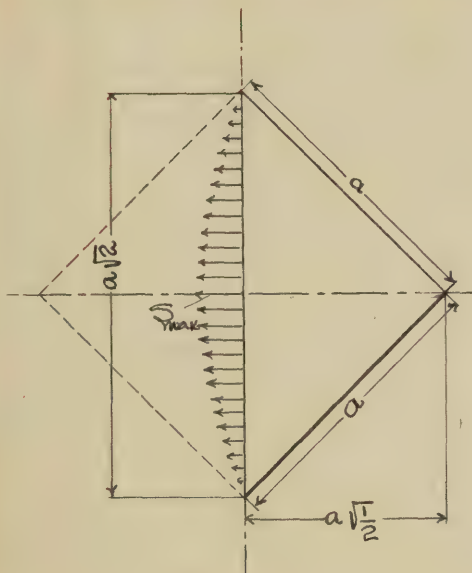


Fig. 1.

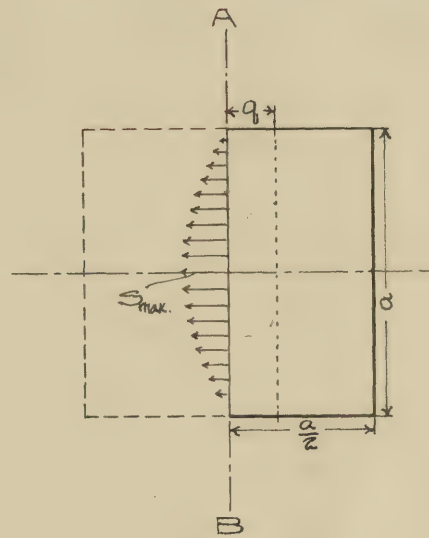


Fig. 2.

Figure 1 shows a square plate with a section taken perpendicular to its surface along a diagonal. The reactions at any point along any side may be considered to be arranged symmetrically, in magnitude, about the center of the side; and the resultant



of the reactions along one side is seen to pass through the middle of the side, from reasons of symmetry. Each side supports a quarter of the load, so that the resultant of the reactions of the two sides shown is  $\frac{1}{2} a^2 p$  in magnitude, and is applied at a distance  $\frac{1}{2} a \sqrt{\frac{1}{2}}$  from the diagonal section. The only other force acting on this side of the diagonal is the resultant of the pressure on half the plate, which is also  $\frac{1}{2} a^2 p$  in magnitude, but is applied at the center of gravity of the triangular area, or at a distance of  $\frac{1}{3} a \sqrt{\frac{1}{2}}$  from the diagonal section. The moment about the diagonal section of all the forces on one side of this section is then:

$$M = \frac{1}{2} a^2 p \cdot \frac{1}{2} a \sqrt{\frac{1}{2}} - \frac{1}{2} a^2 p \cdot \frac{1}{3} a \sqrt{\frac{1}{2}} = \frac{1}{12} a^3 p \sqrt{\frac{1}{2}}.$$

Considering the plate to be a beam, and assuming that the stress perpendicular to the





diagonal section is uniform across the plate,  
we have  $M = \frac{S}{2} I = \frac{S a b^3}{6}$  in which  $b = a \sqrt{2}$   
and  $d = h$ ; hence

$$\frac{1}{2} a^3 p \sqrt{\frac{1}{2}} = \frac{S a \sqrt{2} h^3}{6} \quad \therefore S = \frac{1}{4} \left( \frac{a}{h} \right)^2 p.$$

If the stress is not uniform over the diagonal section, we can call the stress at the center  
=  $k$  x the mean stress ( $k$  is an unknown constant)  
and  $M = S_{\text{max}} \frac{I}{2} = \frac{S_{\text{max}} I}{k}$ ; this gives us

$$S_{\text{max}} = \frac{1}{4} k \left( \frac{a}{h} \right)^2 p \quad \text{for the maximum}$$

stress, which occurs at the center of the plate.

This is only the "apparent stress" as L. P. Murnaghan calls it, and the true resultant stress at the center will bear a definite ratio to this stress, so that we have deduced the form of our equation, but must insert another constant to complete it, obtaining

$$k_b \cong \frac{1}{4} k \left( \frac{a}{h} \right)^2 p, \text{ as stated before.}$$





The true stress is  $T = (1 - \epsilon)S$ , according to the Theory of Elasticity (see Maxima-Mechanics of Materials, "Engineering The Strength of Materials" and Bioton's "Mathematical Theory of Elasticity"),  $\epsilon$  being the coefficient of lateral contraction of cast iron, usually takes as  $\frac{1}{4}$ . If  $E$ , the tensile or compression modulus of elasticity, or Young's Modulus; and  $G$ , the shearing modulus of elasticity, or the modulus of rigidity, be known,  $\epsilon$  may be found by the relation  $G = \frac{E}{2(1 + \epsilon)}$  (Bearing, p. 19), where  $\epsilon = \frac{l}{\epsilon} = \text{ratio of longitudinal extension to lateral contraction}$ .

Hence, if we know  $k$ , we would have a rational formula:  $T = (1 - \epsilon) \frac{k}{4} \left( \frac{a}{h} \right)^2 p$ .

If the stress varied from a maximum at the center to zero at the corners, according to a straight line law, we should have  $k = \frac{a}{2}$ , or the maximum stress equal to



twice the mean stress, and

$$T = (1 - \epsilon) \frac{1}{k} \left( \frac{a}{h} \right)^2 p = \frac{3}{8} \left( \frac{a}{h} \right)^2 p$$

which gives  $C = \sqrt{\frac{3}{2}} = 1.5$ . If the stress were uniform all over the diagonal section, we should have  $k=1$ , and

$$T = \frac{3}{16} \left( \frac{a}{h} \right)^2 p; \text{ or } C = \sqrt{\frac{3}{16}} = .75.$$

This quantity which I have called  $k$  does not appear in Rankine, only the first part of the discussion being taken from his work.

These speculations are not very profitable, since the elastic laws do not hold above the elastic limit, and a formula obtained by these processes will be neither proved nor disproved by tests of the breaking strength of plates. As a matter of interest, however, and so tending to show that the stress at right angles to a diagonal section of the plate varies from zero at the





comes to twice the mean stress at the center, let us apply Bachi's method to round plates; assuming also that  $k = 2$ .

The center of gravity of a semi-circular ring of infinitely small thickness is  $\frac{2r}{\pi}$  from the diameter. The center of gravity of a semi-circular area is  $\frac{4r}{3\pi}$  from the diameter. If a round plate cut along a diameter is imagined, half the load on the plate multiplied by the difference of the above distances will give the moment about the diametrical section; as in the case of square plates.

$$M = \frac{\pi r^3}{2} F \left( \frac{2r}{\pi} - \frac{4r}{3\pi} \right) = \frac{\pi^3}{9} F r^3, \text{ where}$$

$r$  = radius of plate.

$$M = \frac{3F}{8} = \frac{S_{max}}{2} \frac{b d^2}{6} = \frac{S_{max}}{2} \frac{2r h^2}{10};$$

$$\text{hence } S = \frac{2i^3 F}{h^2}, \text{ and } T = (1 - e) = 2(1 - e) \rho \frac{r^2}{h^2}.$$





This is the formula given by Merriman, p. 324, as found by other methods. He bases his practical formula on this.

Since the stresses at right angles to two diagonals, at the center of a square plate, are equal to each other, we have the true stresses in two directions perpendicular to each other, of the same magnitude; and when this is the case, it is shown by the Theory of Elasticity that the "Ellipsoid of Stress" is of circular cross-section in the plane of these two stresses, and the stress in every direction at the center of the plate is the same in magnitude.

If the resultant of the reactions, which lie between the center of one side of a square plate and the corner, be considered to pass through a point distant  $q$  from the center line of the plate, let us find  $q$ , to get some



idea of the distribution of the reactions of the supports. If the same distribution of stress be imagined in a center line (parallel to a side) as on the diagonal — that is, if a parabolic law governs one, assume that it governs the other section — the moment about the center line  $AB$  (see Fig. 2) is

$$M' = q \left[ \frac{a^3}{4} P + \frac{a}{2} \cdot \frac{a^2}{4} P - \frac{a}{4} \cdot \frac{a^2}{2} P \right] = q \frac{a^3}{4} P;$$

$$M' = S_{max} \frac{I d^2}{6} = S_{max} \frac{a h^2}{6}; \quad \text{or,}$$

$$S_{max} = \frac{6 k q a^3 P}{4 a h^2}$$

This is equal to the  $S_{max}$  perpendicular to the diagonal section, found before, on  $p$ .  $\therefore = \frac{k M c}{I} = \frac{k}{4} \left( \frac{a}{h} \right)^2 P$ .

$$\text{Hence } \frac{6 k q a^3 P}{4 a h^2} = \frac{k a^2 P}{4 h^2}; \quad \text{or } q = \frac{2}{6}.$$

Therefore, the resultant of the reactions on the portion of one side extending from the center of the side to the corner of the plate is





applied at one-third of the way from the center of the side to the corner, showing that the reactions probably vary in magnitude from zero at the corner to a maximum at the middle of the side.

The following sketch, copied from one in *Science*, shows how the reactions are great in the middle of the side, and may be zero at the corners, due to the tendency of the deformed plate to approach a spherical shape.



If the straight line law holds for the reactions, it would not be unreasonable to assume that it governs the distribution of stress over a section through the center of the plate.



### Means of Testing.

The apparatus consisted at first of a pump; a plunger carrying a knife-edge on which rested a lever, for measuring the pressure by hanging weights to the end of the lever; and the case in which the plates are placed. This is in two parts, a heavy base and a cover.

The plate to be broken is bolted to the under side of the cover, and the cover is then bolted to the base. The base is shaped to fit the cover, and not to touch the plate tested; in the empty space below the plate water is pumped, and the plate is broken under the pressure.

The apparatus is fully described, with drawings, in the Thesis of Mr. H.C. Bully 1901 who designed it.

To this was added a deflectometer for measuring the deflection at the center of the plate, when bulged upward by the pressure. It is simply a wooden beam resting on two





points in a line at right angles to its length, which rest in center-punch marks in plates fastened to the frame of the apparatus; and on a third point which is on a vertical spindle resting on the center of the plate. A small weight was moved along the short end of the beam until the center of gravity of the combination was brought inside the triangle formed by the three points of support; this insured the beams remaining in equilibrium, supported by all the points. The long end of the beam carries a wire bent at right angles to the length of the beam, and arranged to travel in front of a vertical scale. The distance between the top of the movable spindle, and the line connecting the two fixed points, is one inch; the distance from this line to the end of the wire is fifty inches, so that the deflector multiplies the deflection in the ratio of fifty to one. The fixed points are the sharpened points of



screws which may be removed up and down, to alter the height of the beam, to suit different plates.

The deflectometer could not have been concurrently used with the arrangement for measuring the pressure which was depended on at first. A large hydraulic gauge was fitted however, and this provided a means for instantly reading the pressure.

The only means of calibrating the gauge was the plunger and lever mentioned; the plunger was withdrawn and measured, and the dimensions and weight of the beam found. By putting weights in a bucket hung from the end of the lever, until the gauge read a certain pressure, the pressure could be found by dividing the weight on the plunger by its area, thus giving an absolute method of finding the pressure corresponding to each gauge reading. If the gauge and the plunger be considered equal





accurate, it would not be good practice to rely on the maker's marks without calibrating the gauge.

The plunger may be turned on its axis by a wheel at the top, thus getting rid of most of the friction opposing its vertical travel. A step-bearing for the knife edge, resting on balls, holds the core. The combination forms a means of measuring the pressure which is probably accurate - it is not unlike a Crosby Gauge Tester in principle - but it is not convenient to use, since the pressure has to be kept constant by continually moving the pump handle, and it is difficult to turn the plunger at the same time. By calibrating the gauge, it could be depended on alone for the pressure, and two observers could take all the readings necessary. One could work the pump and keep the pressure as nearly constant as possible.



while the other took readings on the deflection meter scale.

The same pump and gauge were used by Messrs. Good and Medford in their train work, so that it was necessary to put a stop valve in the pipe leading to their apparatus, and one in the connection between the gauge and the apparatus for these experiments. In addition to this, a drain valve was put in between the stop valve and the testing machine, so that the pressure could be released by opening this valve to the atmosphere.

Photograph No. 1 shows the edge of the testing machine on the left; then, in order: the drain valve; the stop valve; the gauge; the calibrating lever, and wheel for turning the plunger as described; and the pump at the back. To the right of the gauge is the stop valve for shutting off







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communication with the apparatus of Messrs. Wood and Redfield. Their work required pressures of over 2000 pounds; and to diminish the risk of injury from explosion, they encased part of the apparatus in wood. The water for the pump was taken from a rubber tube running to the bucket shown at the back.

Photograph No. 2 shows the apparatus set up for a test. The top of the plate tested was in the plane of the joint between the base and cover of the testing machine. The cover was screwed down by getting a purchase on the large nuts at the top, with an engine wrench and a length of pipe; and to preserve the table from collapse, Mr. Wood gave valuable assistance by putting in the bracing seen below the top. In order to handle the heavy upper casting alone, I hung a hoist above the machine.









### The Tests.

After cutting ripples and fitting the pipe connections between the machines and the pump, the first plate was bolted to the cover. This joint was made in all the tests by placing a gasket of yellow "detail" paper between the cover and the plate, and packing both ends of each of the sixteen bolts with lamp wick soaked in oil and red lead. The half-inch bolts provided for this joint are too small (as are the one-inch bolts of the outside joint), and in order to make the joint tight, each nut had to be screwed down until the limit of elasticity of the material was about reached. Nearly a dozen half-inch bolts were broken in the course of the work, and even then there was trouble from water leaking between the bolt-hole and bolt. The paper gasket about





unavoidably prevented leakage between the flat surfaces of the plate and cover, this was due to the fact that the pressure on the under side of the plate forced it up against the cover.

The outside joint gave more trouble. The load on the plate tended to force the cover off the base, and at a fairly high pressure the joint would commence to leak.

This joint was made tight at first by placing a gasket of yellow paper between the faces, extending from the inside to the outside edge, and then wrapping wire-yarn coated in oil and red lead around each large bolt.

This took a long time and was not very satisfactory, but served for breaking plate No. 1.

The gage and deflectionmeter were fitted before testing the second plate. This plate was the thickest one tried, and gave a great deal of trouble. Different authors



of packing were tried; strip solder laid just inside the large bolts was suggested by Prof. Silveira, and gave good results; but after carrying a heavy pressure and then releasing it, the bolts would all have to be tightened down again, which would destroy the adjustment of the deflectometer. The best arrangement was found to be a gasket of two layers of yellow paper extending from the inside edge of the face to the bolts. After many readjustments the pressure was carried to fifteen hundred pounds. Instead of the plate breaking, however, the cover of the testing machine started to crack at the top in each corner, and it was judged wise to discontinue the attempts to break plate No. 2.

Several of the large bolts were broken in screwing down the nuts, in adjusting this and other plates. One bolt





broke off below the surface, and the latter plant was dismantled, the lower casting of the machine being taken down in the shop, where the end of the bolt was removed.

One plate was broken with one bolt missing, as the shop could not turn out bolts at the rate they were broken.

An attempt was made to arrange a plate so that there would be no bending moment or restraint due to bolts around the edge, so that it might be considered simply supported. To do this the plate was placed between two frames of copper wire bent into equal squares, and placed one vertically above the other. A copper and an iron wire were also used. The joint could not be made tight, however, and finally the edge of the plate buckled under the pressure caused by screwing down the nuts.



A ribbed plate was designed, to see what effect ribbing would have on the strength and stiffness of the plate.

The ribs were run parallel to the sides and designed as beams of uniform strength. Although the patterns were not made in exact accord with the design, the plates answered their purpose very well.

Photograph No. 2 shows one of them on the table. One plate was broken with the ribs on the tension side, and near the end of the time another was ordered from the same pattern, and the ribs put on the compression side. The ribs were cut off near the edges of the plate to allow the edges to be planed up for the joint; and when the ribs were put on the lower side there was an unsupported portion of the plate between the end of the rib and the joint. The break occurred in the





portion, and not in the rib, in the case of the last plate.

To give some idea of the quality of the cast iron composing the plates, specimens of the material of the first lot of plates were cut from an extra plate made at the same time. Seven specimens were tested in tension, one in direct tension, and the balance afterwards broken, and two were tested in tension. Owing to the small size of the specimens, the extensometer could not be used for finding Young's Modulus, and instead of this,千分尺 measurements were taken of the deflections of the beams tested. Two tension tests were made of the material from the second set of plates. The last plate was cast by itself, a little while after the second lot, and no specimens of the material were obtained. The castings were fairly uniform, except Plate No. 5.



## Results.

The thickness of each plate was measured by taking readings with calipers and steel rule over a square inch area at the center of the plate, as this is the region where the crack probably starts.

The pressure is taken from the gauge calibration.

The strength of the material was obtained from the following data.

The tension specimens were  $3\frac{1}{16}$ " long, with squared ends  $\frac{5}{16}$ " on a side, the center portion being a cylinder  $\frac{1}{2}$ " in diameter.

The Thurston tensile machine was calibrated with the bob removed from the lever, and the ends of the specimens were inserted in the square holes of blocks provided for such pieces.

With the arm of the machine locked, its moment about the axis was found to be 1447 inch-pounds, corresponding to a height "q" of  $\pm .27$  on the autographic diagram.





Hence, at any point of a curve the ordinate multiplied by  $\frac{1447}{4.27}$  gives the torsional moment  $P\rho$  on the piece. The abscissa of the curve in inches, multiplied by 10, gives the angular deflection in degrees, since the drum is 36" in circumference. From the curve of specimen No. 1,  $y = 1.078$  at the breaking point. The ultimate shearing strength is then

$$S = \frac{P\rho c}{J} = \frac{P\rho \times 10}{\pi d^3} = \frac{\frac{1447}{4.27} \times 1.078 \times 10}{\pi (4.851)^3}$$

4851" being the diameter of the piece at the part fractured. Hence:

$$S = 25,550 \text{ pounds per square inch}$$

Drawing a straight line coinciding with the direction of the strain curve above the elastic limit, where it is very nearly straight, the tangent of the inclination of this line  $= \frac{y}{x} = \frac{5.18}{2.0}$ . To the proper scale this becomes  $\frac{8.35 \times \frac{1447}{4.27}}{2 \times 10} = \frac{P\rho}{\pi}$ .



The torsional rigidity (modulus of rigidity)  $G$  is hence:

$$G = \frac{2S_e L}{\theta d} = \frac{32 P \theta L}{\pi \theta d^4} = \frac{32 P L}{\pi \times \frac{\pi}{150} d^4}$$

where  $L$  = length of piece between blocks.

$$G = \frac{32 \times 8.35 \times 1447 \times 1.719 \times 9}{\pi^2 \times 4.27 \times (.4851)^4} = 2,563,300.$$

In the same way the second torsion specimen gave:

$$S_e = \frac{1447 \times 1.66 \times 16}{4.27 \times \pi \times (.5027)^3} = 22,550.$$

$$G = \frac{32 \times 1447 \times 1.719 \times 1^2 \times 11.25}{\pi^2 \times 4.27 \times (.5027)^4 \times 3} = 1,196,000.$$

The third specimen gave:

$$S_e = \frac{1447 \times 2.07 \times 16}{4.27 \pi (.5184)^3} = 25,650.$$

The fourth specimen gave:

$$S_e = 25,500$$





The mean of these values is:

$$\begin{array}{r} 25,550 \\ 25,550 \\ 25,550 \\ 25,510 \\ \hline 104,210 \end{array}$$

$S_u = 24,815$  for the

ultimate shearing strength of the material of the first lot of plates.

The two specimens broken as beams gave the following modulus of rupture ( $S_r$ ):

$$S_r = \frac{M_c}{I} = \frac{W}{b} \cdot \frac{l}{h^2} = \frac{6 \times 8.969 \times 1080}{4 \times 6.515 \times (7.883)^2}$$

$= 35,890$ ; where the span of the beam,  $l$ , was 8.969"; the breaking load  $W = 1080$  pounds; the width of the beam  $= 6.515$  " and the depth  $= 7.883$  ".

In the case of the other specimen.

$$S_r = \frac{6 \times 735 \times 8.969}{4 \times 6.262 \times (6.297)^2} = 39,826.$$

The mean of these is 37,855 pounds per square inch.



For the pieces broken in direct tension, the only reliable value for the first material was given by a piece .6182" x .7475", broken at 7.650 pounds, giving an ultimate strength

$$S_{ult} = \frac{7650}{.6182 \times .7473} = 16,560 \text{ pounds per sq. in.}$$

The two half pieces which had been previously stressed, gave

$$S_{ult} = \frac{7050}{.7739 \times .6390} = 14,200. \quad \text{and}$$

$$S_{ult} = \frac{7410}{.6268 \times .6135} = 19,310. \quad \text{The}$$

breaks occurred in the jaws, however, and these values will not be used. The mean

of these is

$$\begin{array}{r} 14200 \\ 19310 \\ \hline 33510 \\ 2 \quad \hline 16755 \end{array}$$

$$S_{ult} = 16,755$$

which is not

very different from the value adopted:

$$S_{ult} = 16,500 \text{ pounds per sq. in.}$$

The two specimens of the material of the second lot of plates were broken in direct tension



with the following results:

The smaller specimen gave

$$S_{ult} = \frac{6460}{.3369 \times 1.065} = 18,005.$$

The larger specimen gave

$$S_{ult} = \frac{8700}{.3599 \times 1.1725} = 20,617.$$

The mean of these is:

$$S_{ult} = 19,311 \text{ pounds per sq. in.}$$

If the modulus of rupture  $S_r$ , obtained by breaking a specimen of the material as a beam, be inserted for  $S$  in the equation

$h = C a \sqrt{\frac{P}{S}}$ , and the corrected breaking pressure be put for  $p$ , solving for  $C$ , we get for plates No. 1, 3, and 5 respectively

$$C = \frac{h}{a} \sqrt{\frac{S}{p}} = 4428$$

$$C = 4434$$

$$C = 4479$$

The value of this constant is given by Baer for





No. of Plate	1	2	3	4	5	6	7	8	9
Thickness of center, = $t_c$	5156"	8008	6094	$\frac{3}{8}$ "	3999	5109	6350	net = <del>2537</del> 1.1022	net = <del>4375</del> 1.0000
Length of side = $a$	7.51"	7.51	7.51	<del>7.51</del> 11.75	7.51	7.51	7.51	7.51	7.51
Breaking Force, $P_{avg}$	—	—	1350	—	540	477	1100	500	920
Breaking Force, $P_{max}$	910	—	1327	—	535	950	1077	489.5	890
Ultimate Tensile Strength of Mat.	16,560	16,720	16,540	19311	16,540	19311	19311	19311	P. 2.24
Ultimate Strength of Mat.	37,855	37855	37855	—	37855	—	—	—	—
Ultimate Shear Strength of Mat.	24,815	24815	24815	—	24815	—	—	—	—
$E$ Mod. of Mat.	—	—	—	—	—	—	—	—	—
$I$ Mod. of Replicity	2,280,000	2,280,000	2,280,000	2,280,000	2,280,000	—	—	—	—
$C$	4428	—	4334	—	4479	—	—	—	—
$C'$	2929	<del>2967</del>	2867	—	2963	3067	3504	—	—

TABLE OF RESULTS.



fixed plates, as 433, as shown on page 8.  
The mean of the values found from these  
experiments is

$$C = .4414$$

The values obtained by Mr. Kelly, as  
stated in his thesis - "The Strength of Flat Plates",  
1901, are  $C = .3, .44, \text{ and } .43$ .

Putting the mean  $C$  in the formula, we  
have

$$h = .4414 a \sqrt{\frac{P}{S}}$$
 This may be  
used as a working formula by putting  
for  $S$  the modulus of rupture of the material,  
well known, divided by a suitable factor of  
safety.

In practice, the tensile strength of  
the material is usually known, or is easy to  
obtain, and a more useful formula than the  
one given above, may be found by finding the  
constant which will satisfy the equation when  
the ultimate tensile strength is inserted for  $S$ .





Since the ultimate tensile strength of the second set of plates was found, we can obtain a value for this constant for five plates. It is called  $C'$ , and the values are given in the table. The mean of the five values is

$$C' = .3070 \quad \text{This gives us}$$

the working formula:

$$h = .3070 a \sqrt{\frac{E}{S}}$$
 in which  $S$  represents the ultimate tensile strength of the material, divided by a factor of safety. This is the formula which I offer, to be used in designing square plates held by bolts.

Applying this equation to the two ribbed plates, let us find the pressure they probably would have supported if they had been flat plates without ribs, and <sup>having</sup> the thickness of the webs of plates nine and eight.

Plate No. 8 had ribs on the tension side;



the total thickness was 1.047", and the web alone was .4687" thick. A flat plate .4687" thick should stand a pressure of

$$P = \frac{5 h^2}{E t^2 a^2} = \frac{19311 \times (.4687)^2}{(.307)^2 \times (7.51)^2} = 798 \text{ pounds.}$$

This plate actually stood only 789.5 pounds. Hence, it was considerably weakened by putting on the extra metal in the shape of ribs. This may be explained by the fact that the distance from the neutral surface to the uppermost fibre of the rib is so great that this fibre breaks under a small load, since the stress is proportional to the distance from the neutral surface. After this fibre breaks, those below it give way, until the rib is broken as far as the web of the plate. The plate is now in the condition of having stiffening ribs running from the center to the sides, but not supported at the center. The radius of curvature of the bent plate is very small at the center, and the plate breaks



under a load smaller than it would have carried if it had been unribbed.

The plate (No. 7) with the ribs on the compression side, was 1.0625" thick over all; the web alone being .375" thick. Without any ribs, it would have stood a pressure of

$$p = 17311 \left( \frac{.375}{.307 \times 1.51} \right)^2 = 695.3 \text{ pounds.}$$

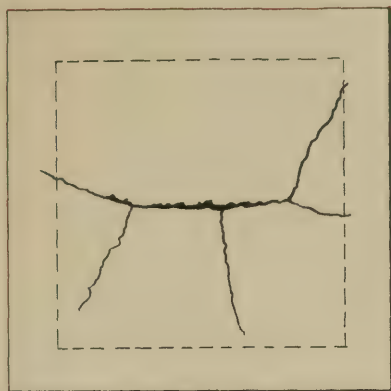
It actually stood 890 pounds, and would have stood more if the ribs had been longer. Therefore, it is bad practice to put ribs on the unloaded or tension side of a cast iron plate; but they may be put on the loaded side with advantage; since the compressive strength of cast iron is much greater than its tensile strength.

The ribs should be run parallel to the sides, since this gives the shortest span for the rib, considered as a beam.

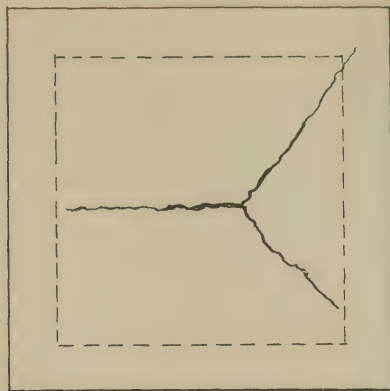
done  
in 1890  
J. H. P.



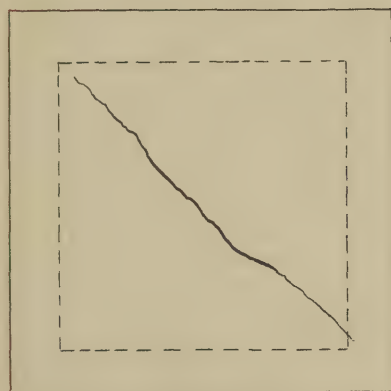




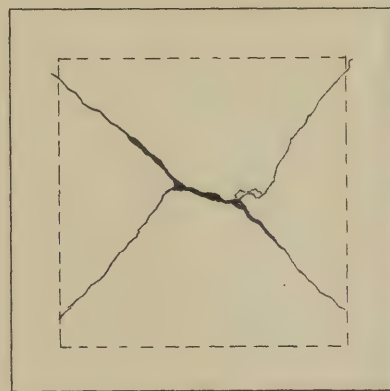
No. 1



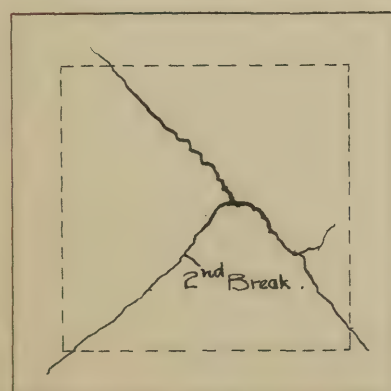
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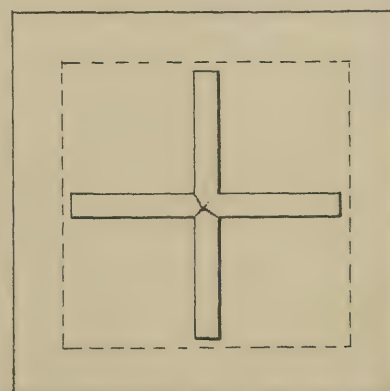
No. 5



No. 6.



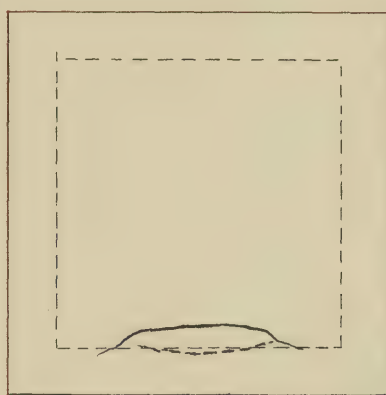
No. 7.



No. 8.

APPEARANCE OF CRACKED PLATES.



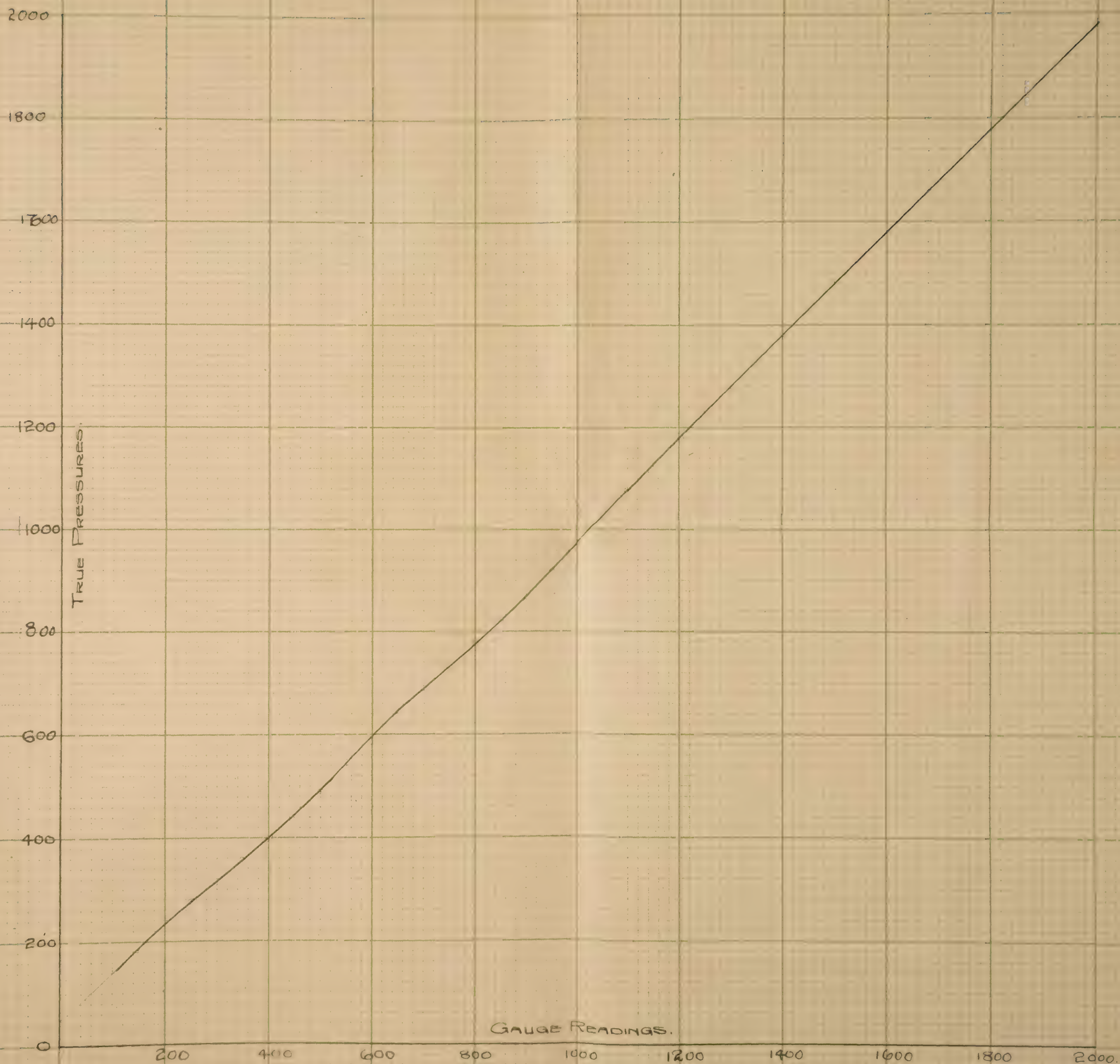


No. 9.

The dotted line shows the inside edge of the cover, in each sketch.



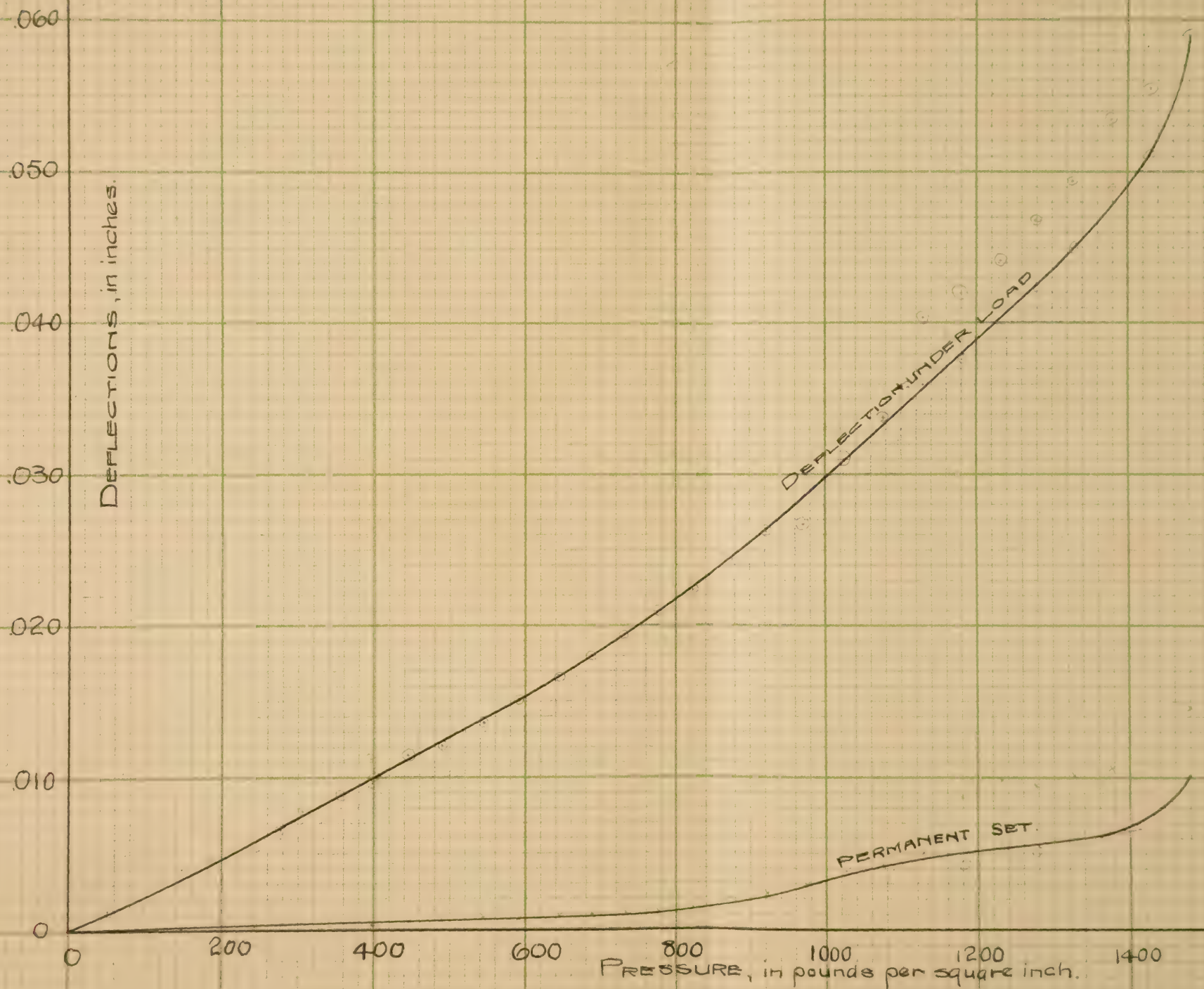




CALIBRATION CURVE OF HYDRAULIC PRESSURE GAUGE, No. 28046.







STRAIN DIAGRAM - PLATE NO. 2.



0.060

0.050

0.040

0.030

0.020

0.010

0

DEFLECTIONS, in inches

DEFLECTIONS UNDER LOAD

PERMANENT SET

0

200

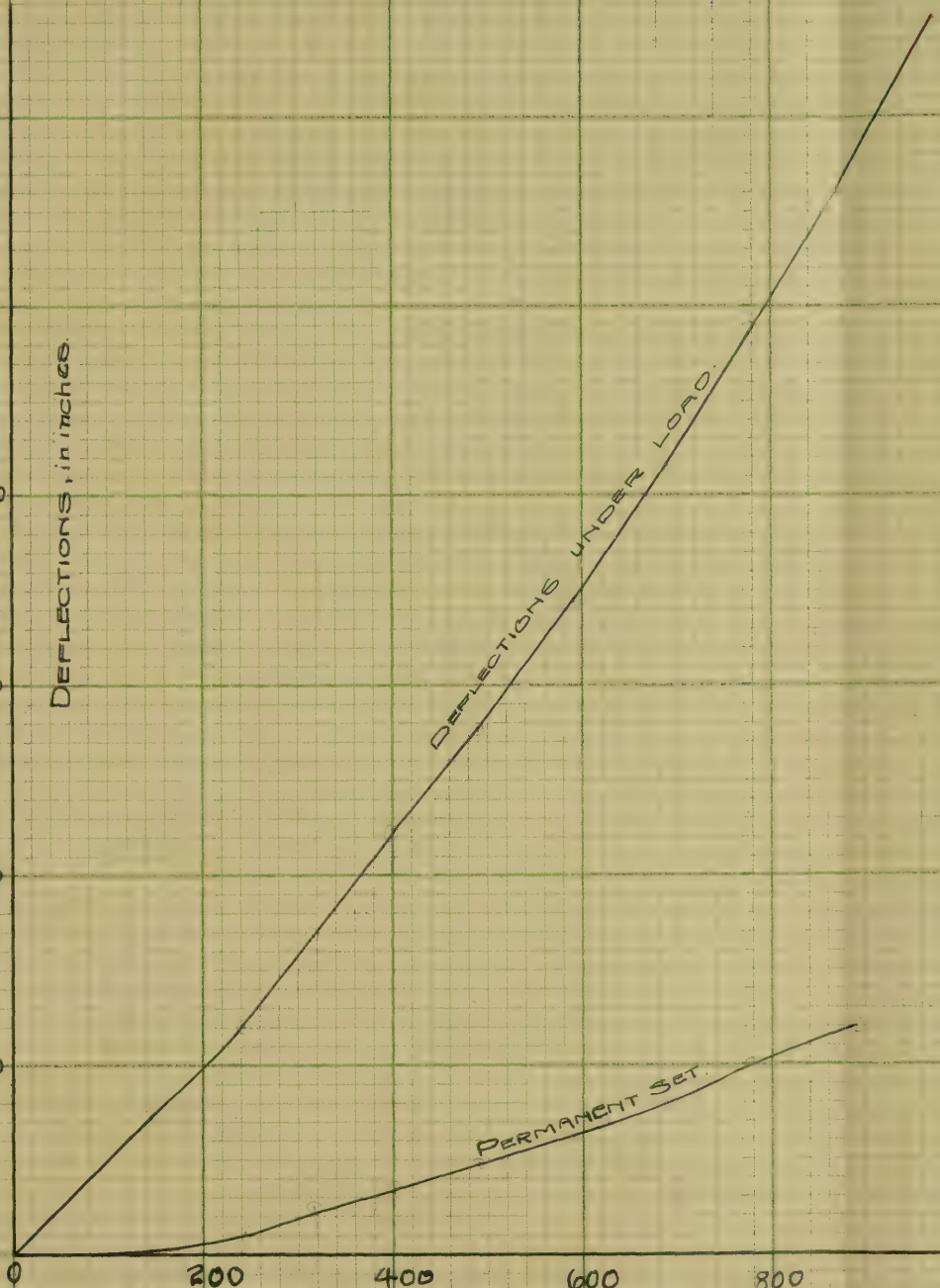
400

600

800

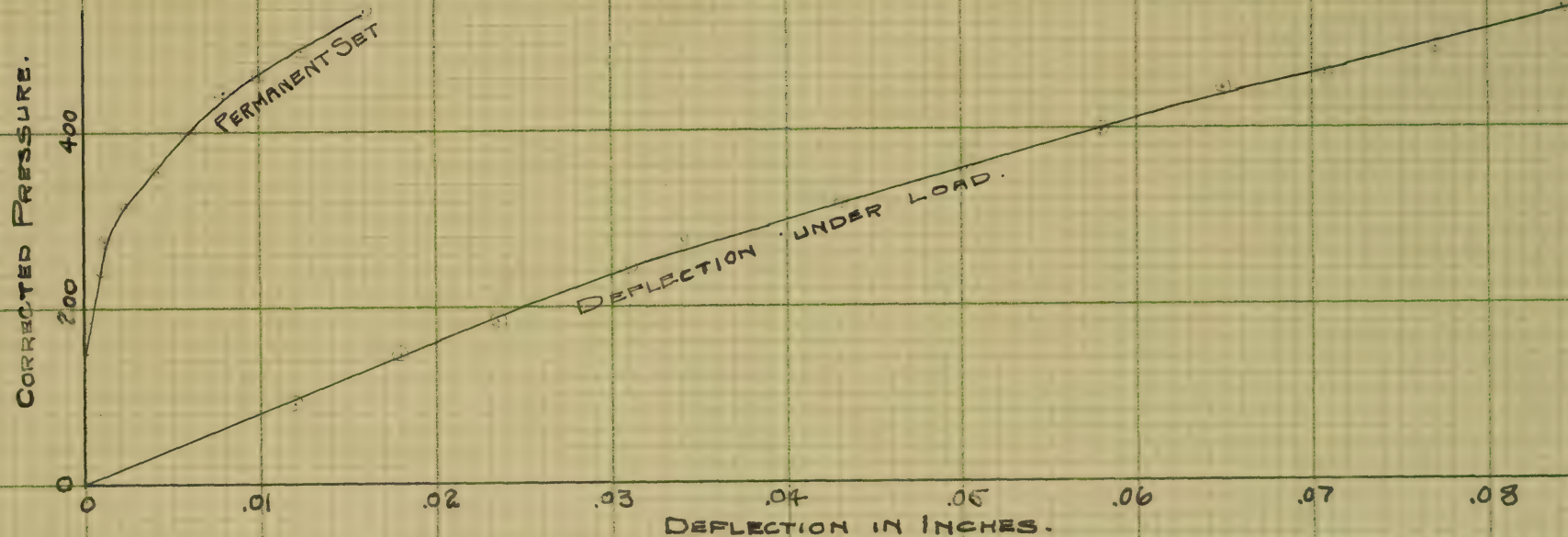
1000

PRESSURE, in pounds per square inch.  
STRAIN DIAGRAM - PLATE No. 3.









STRAIN DIAGRAM OF PLATE No. 5.



Typical strain diagrams, derived from the deflectionometer readings, are shown, taken from plates No. 2, 3, and 5. These exhibit no decided bend, doubtless because cast iron has no definite elastic limit. The permanent sets given, are those remaining after the loads, corresponding to these sets in the diagram, were removed. These also follow a flat curve in most of the plates.

In conclusion, I wish to acknowledge my indebtedness to Mr. Greene for the assistance received throughout the course of the work.

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### Note.

The deflections of the two specimens of the material of the first lot of plates, which were broken as beams, were found by taking observations of the height of a mark at the middle of the beam with a Kathetometer.

The amount the knife-edge of the Olsen Testing Machine moved, was also calculated by counting the number of turns of the gears.

The two sets of values were plotted, but the strain diagrams thus obtained do not agree at all closely, and are very irregular in shape, so that it was judged that no value of Young's Modulus based on these curves would truly represent the quality of the material.







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